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## Semiclassical scattering amplitudes of dressed gravitons

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### Abstract

We consider effective action for the Einstein gravity and show that dressed mean fields are actual variables of the effective action. Kernels of this effective action expressed in terms of dressed effective fields are constituent parts of scattering amplitudes for gravitons. Possible applications to the graviton scattering and black hole formation are discussed at the semiclassical level. In particular, we consider graviton scattering in four dimensions based on the Lipatov effective action for quantum gravity, shock waves of particles moving on the brane in Randall-Sundrum scenario with fifth extra dimension, and Giddings' estimation of Froissart bound.

Keywords: Einstein gravity, 1PI effective action, Slavnov–Taylor identity, eikonal amplitude

# 1 Introduction

There is an intriguing belief that the high energy scattering in gauge theories in four space-time dimensions can be described by the bulk physics of supergravity with a higher dimension in AdS space. In other words, one can extract information about quantum process amplitudes in four spacetime dimensions starting with the classical physics of wider theories including gravity in higher dimensions. This is called gauge/string-gravity duality and is based on Maldacena conjecture [1].

The total cross section for the particle absorption by a black hole is estimated by the area of the black hole horizon in Ref. [2] for four dimensions in a semiclassical way. Another estimation has been given in Ref.[3]. In Ref. [5] it has been shown that the horizon is necessarily formed in three dimensions at sufficiently high energies when two particles collide. The amplitude of this process has been calculated at the semiclassical level in Ref. [4], in which classical Hamiltonian can be found.

In four dimensions there are no exact results about the cross section of the process *two particles*  $\rightarrow$  *black hole* even at the classical level. One has to rely on approximate estimates such as in Ref. [2, 3]. In unrestricted four-dimensional spacetime the horizon area is proportional to the square of total energy in center-mass frame. However, in brane models as it has been shown by Giddings [6] radius of horizon does modify its form to logarithm of c.m. energy. This is the basic source of the Froissart bound (FB) for the total cross sections which has been known since 1961 as a consequence of unitarity, positivity of the imaginary part of partial wave amplitudes of analytic S-matrix [7]. Giddings [6] concluded that bulk theory feels in some way unitarity of the four-dimensional theory on the boundary.

At the same time there is approach to consider the effective action of quantum gravity as a functional of the effective field of metric convoluted with unspecified dressing function [8, 9, 10, 11]. We will call this construction “dressed effective field”. In this paper we make steps to embed the results of semiclassical and eikonal estimations of the amplitudes *two particles*  $\rightarrow$  *black hole* process and graviton scattering processes into the approach of Ref. [9, 11, 16].

The effective action for quantum gravity is Legendre transform of the logarithm of path integral. Tilded fields in this paper will mean dressed effective fields  $\tilde{h}$  and  $\tilde{\phi}$  of graviton and matter fields, that is the effective fields convoluted with unspecified dressing function [8, 9, 10, 11],

$$\begin{aligned}\tilde{h}_{\mu\nu}(x) &= \int d^D x G_h^{-1}(x - x') h_{\mu\nu}(x') \\ \tilde{\phi}(x) &= \int d^D x G_\phi^{-1}(x - x') \phi(x'),\end{aligned}$$

$h_{\mu\nu}$  is the graviton tensor,  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , where  $\eta_{\mu\nu}$  is Minkowski tensor.

In addition to analyzing size of horizon along the lines of Ref. [6] we consider graviton scattering in four dimensions based on the Lipatov effective action for quantum gravity [18], scattering of one particle by a shock wave of another one both of which move on on the brane in Randall-Sundrum scenario with fifth extra dimension. We start our analysis with introducing the concept of the dressed effective fields.

## 2 Effective action of dressed gravitons in four dimensions

There is a way to write a functional structure of the effective action for dressed gravitons in supergravity [11]. For example, one can consider supergravity in the component formulation. Strictly speaking, if one works with supergravity the vielbeins must be introduced. Since we do not want to overload notation in this paper we work with the metric as an independent field which is variable of integration in the path integral. This would be sufficient for the purpose of the present paper. To clarify the idea we elaborate usual four-dimensional Einstein gravity as an example in this section.

Let us take the path integral for the theory of four dimensional gravity in the following form [14]:

$$Z = \int dg d\phi \exp i [S[g, \phi]],$$

where  $S[g, \phi]$  is the classical action of D4 gravity  $g$  coupled to matter fields  $\phi$ . The action of the gravitational field is usually taken to be

$$S = \frac{1}{16\pi} \int R \sqrt{-g} d^4x.$$

In such approach we can fix the symmetry of the diffeomorphism group by imposing some linear gauge fixing condition on the graviton field  $g_{\mu\nu}$ . One can take the most general form of a linear gauge fixing condition

$$F[g_{\mu\nu}] = 0.$$

To fix the gauge we have to introduce into the path integral the gauge fixing term and Faddeev–Popov ghost field at the same time in order to factorize the volume of diffeomorphism out [12]. This procedure brings out additional symmetry for the classical action extended by gauge fixing term and by the ghost term which is called BRST symmetry [13]. Total action including gauge fixing, FP ghost action, at the classical level can be written as

$$\begin{aligned} S &= S_{\text{cl}} + S_{\text{gf}} + S_{\text{gh}} \\ &= \frac{1}{16\pi} \int d^4x R \sqrt{-g} - \int d^4x \frac{1}{2\alpha} (F[g_{\mu\nu}])^2 \\ &\quad - \int d^4x i b \frac{\delta F}{\delta g_{\mu\nu}} \mathcal{L}_c g_{\mu\nu} + S_I(g_{\mu\nu}, \phi), \end{aligned} \quad (1)$$

where  $c^\mu(x)$  here is ghost field,  $b(x)$  is antighost field, and  $\mathcal{L}_c$  is Lie derivative associated with ghost field  $c^\mu(x)$  that acts on the metric field as

$$\mathcal{L}_c g^{\mu\nu} = c^\lambda \partial_\lambda g^{\mu\nu} - (\partial_\lambda c^\mu) g^{\nu\lambda} - (\partial_\lambda c^\nu) g^{\mu\lambda},$$

and  $S_I(g, \phi)$  is the action term containing the interaction between gravity and matter. The BRST symmetry for the action (1) can be written as

$$\begin{aligned} g_{\mu\nu} &\rightarrow g_{\mu\nu} + i \mathcal{L}_c g_{\mu\nu} \varepsilon, \\ \phi &\rightarrow \phi + i \mathcal{L}_c \phi \varepsilon, \\ c &\rightarrow c - \frac{1}{2} \mathcal{L}_c c \varepsilon, \\ b &\rightarrow b + \frac{1}{\alpha} F \varepsilon, \end{aligned} \quad (2)$$

$\varepsilon$  is Grassmannian constant parameter of the BRST symmetry,  $\varepsilon^2 = 0$ . This invariance at the level of quantum theory can be transformed in a usual way [12] to Slavnov–Taylor (ST) identity that is the equation for the Legendre transform of the logarithm of the path integral. This Legendre transform is performed with respect to external sources of the theory which are coupled in the path integral to the quantum fields from the measure of the path integral. To do this one defines the path integral extended by the dependence on the following external sources

$$\begin{aligned} Z[T, \eta, \rho, K, k, L] = & \int dg_{\mu\nu} dc_\lambda db d\phi \exp i [S \\ & + \int d^4x T^{\mu\nu} g_{\mu\nu} + \int d^4x j \phi + i \int d^4x \eta_\mu c^\mu + i \int d^4x \rho b \\ & + i \int d^4x K^{\mu\nu} \mathcal{L}_c g_{\mu\nu} + i \int d^4x k \mathcal{L}_c \phi - \int d^4x L_\mu \frac{1}{2} \mathcal{L}_c c^\mu], \end{aligned} \quad (3)$$

where new external sources  $K^{\mu\nu}$ ,  $k$ , and  $L_\mu$  coupled to the BRST variations of the metric, matter field and the ghost field under group of diffeomorphisms are introduced, respectively. The effective action  $\Gamma$  is related to  $W = i \ln Z$  by the Legendre transformation

$$\begin{aligned} g_{\mu\nu} &\equiv -\frac{\delta W}{\delta T^{\mu\nu}}, \quad \phi \equiv -\frac{\delta W}{\delta j}, \quad ic^\mu \equiv -\frac{\delta W}{\delta \eta_\mu}, \quad ib \equiv -\frac{\delta W}{\delta \rho}, \\ \Gamma &= -W - \int d^4x T^{\mu\nu} g_{\mu\nu} - \int d^4x j \phi - i \int d^4x \eta_\mu c^\mu - i \int d^4x \rho b \end{aligned} \quad (4)$$

If all equations Eq. (4) can be inverted,

$$\begin{aligned} \Omega &= \Omega[\varphi, K^{\mu\nu}, k, L_\mu], \\ \Omega &\equiv (T^{\mu\nu}, j, \eta_\mu, \rho), \quad \varphi \equiv (g_{\mu\nu}, \phi, c^\mu, b). \end{aligned}$$

the effective action can be defined in terms of new variables,  $\Gamma = \Gamma[\varphi, K^{\mu\nu}, k, L_\mu]$ . Hence the following equalities hold:

$$\begin{aligned} \frac{\delta \Gamma}{\delta g_{\mu\nu}} &= -T^{\mu\nu}, \quad \frac{\delta \Gamma}{\delta \phi} = -j, \quad \frac{\delta \Gamma}{\delta K^{\mu\nu}} = -\frac{\delta W}{\delta K^{\mu\nu}}, \\ \frac{\delta \Gamma}{\delta k} &= -\frac{\delta W}{\delta k}, \quad \frac{\delta \Gamma}{\delta c^\mu} = i\eta_\mu, \quad \frac{\delta \Gamma}{\delta b} = i\rho, \quad \frac{\delta \Gamma}{\delta L_\mu} = -\frac{\delta W}{\delta L_\mu}. \end{aligned} \quad (5)$$

If the transformation Eq. (2) is made in the path integral Eq. (3) one obtains (as the result of the invariance of the integral Eq. (3) under a change of variables) the ST identity:

$$\begin{aligned} & \left[ \int d^4x T^{\mu\nu} \frac{\delta}{\delta K^{\mu\nu}} + \int d^4x j \frac{\delta}{\delta k} - \int d^4x i\eta_\mu \left( \frac{\delta}{\delta L_\mu} \right) \right. \\ & \quad \left. + \int d^4x i\rho \frac{1}{\alpha} F \left[ \frac{\delta}{\delta T^{\mu\nu}} \right] \right] W = 0, \end{aligned}$$

or, taking into account the relations Eq. (5), we have

$$\begin{aligned} & \int d^4x \frac{\delta \Gamma}{\delta g_{\mu\nu}} \frac{\delta \Gamma}{\delta K^{\mu\nu}} + \int d^4x \frac{\delta \Gamma}{\delta \phi} \frac{\delta \Gamma}{\delta k} + \int d^4x \frac{\delta \Gamma}{\delta c^\mu} \frac{\delta \Gamma}{\delta L_\mu} \\ & \quad - \int d^4x \frac{\delta \Gamma}{\delta b} \left( \frac{1}{\alpha} F[g_{\mu\nu}] \right) = 0. \end{aligned} \quad (6)$$

In addition to ST identity also there is the ghost equation that can be derived by shifting the antighost field  $b$  by an arbitrary field  $\varepsilon(x)$  in the path integral. The consequence of invariance of the path integral with respect to such a change of variable is (in terms of the variables (4)) [12]

$$\frac{\delta\Gamma}{\delta b} + \frac{\delta F}{\delta g_{\mu\nu}} \frac{\delta\Gamma}{\delta K^{\mu\nu}} = 0. \quad (7)$$

The ghost equation (7) restricts the dependence of  $\Gamma$  on the antighost field  $b$  and the external source  $K_M$  to an arbitrary dependence on the combination

$$\frac{\delta F}{\delta g_{\mu\nu}} b(x) - K^{\mu\nu}(x). \quad (8)$$

The main idea of Refs. [9, 10] is that the momentum-dependent part of the  $Lcc$  correlator related to the superficial divergence (divergent in the limit of removing regularization) is invariant itself with respect to ST identity in each order of the perturbation theory. According to Ref. [9, 16], this invariance results in the following integral equation for the part of the correlator  $Lcc$  corresponding to the superficial divergence  $\sim \ln \frac{p^2}{\Lambda^2}$

$$\begin{aligned} \int dx \Gamma_\Lambda(y', x, z') \Gamma_\Lambda(x, y, z) &= \int dx \Gamma_\Lambda(y', y, x) \Gamma_\Lambda(x, z, z') \\ &= \int dx \Gamma_\Lambda(y', x, z) \Gamma_\Lambda(x, z', y), \end{aligned} \quad (9)$$

where  $\Gamma_\Lambda(x, y, z)$  is the scale-dependent part of the correlator  $Lcc$  corresponding to the superficial divergence in each order of the perturbation theory [16],  $\Lambda$  is a scale of ultraviolet regularization. The most general parametrization of this correlator is  $\Gamma(x, y, z)$ <sup>1</sup>,

$$\Gamma \sim \int dx dy dz \Gamma(x, y, z) L_\mu(x) c^\lambda(y) \partial_\lambda c^\mu. \quad (10)$$

As has been shown in Ref. [9], the only solution to the integral equation (9) is

$$\Gamma_\Lambda(x, y, z) = \int dx' G_c(x' - x) G_c^{-1}(x' - y) G_c^{-1}(x' - z), \quad (11)$$

where  $G_c(x)$  is some unspecified function,  $\int d^4x' G_c(x - x') G_c^{-1}(x' - y) = \delta(x - y)$ . The complete correlator  $Lcc$  can be then written as

$$\begin{aligned} &\int dx dy dz \Gamma(x, y, z) L_\mu(x) c^\lambda(y) \partial_\lambda c^\mu(z) = \\ &= \int dx' dy' dz' dx dy dz \tilde{\Gamma}(x', y', z') G_c(x' - x) G_c^{-1}(y' - y) \times \\ &\quad \times G_c^{-1}(z' - z) L_\mu(x) c^\lambda(y) \partial_\lambda c^\mu(z) \\ &= \int dx' dy' dz' \tilde{\Gamma}(x', y', z') \tilde{L}_\mu(x') \tilde{c}^\lambda(y') \partial_\lambda \tilde{c}^\mu(z'). \end{aligned}$$

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<sup>1</sup>The structure of indices in (10) is appropriate for the part of the correlator  $Lcc$  related to its superficial divergence at least.

Here  $\tilde{\Gamma}(x', y', z')$  is the kernel of  $Lcc$  correlator written in terms of dressed fields  $\tilde{L}_\mu$  and  $\tilde{c}^\mu$  defined by the convolutions

$$\begin{aligned}\tilde{L}_\mu(x) &= \int d^4x' G_c(x - x') L_\mu(x'), \\ \tilde{c}_\mu(x) &= \int d^4x' G_c^{-1}(x - x') c_\mu(x').\end{aligned}$$

As has been shown in Ref. [16], the consequence of such a structure for the  $Lcc$  correlator is that the effective action can be expressed in terms of the dressed effective fields for the rest of proper correlators. The effective action expressed in terms of the dressed effective fields possesses some kernels which in fact are constituent parts of the scattering amplitudes. In  $\mathcal{N} = 4$  super-Yang–Mills theory such kernels do not depend on the ultraviolet regularization scale [16]. It is known for a long time that gravity cannot be renormalized by adding finite number of counterterms to remove all the divergences. One can think that gravity is regularized in some way that conserves symmetry of the classical action, for example by dimensional regularization. These kernels will depend on the UV scale and on mutual distances of  $n$  spacetime coordinates of some  $n$ -point proper Green function. For scattering amplitudes of gravitons in multi-Regge kinematic region of the momentum space these kernels can be calculated in the leading logarithmic approximations as it has been shown in Ref. [18]. We will use this action in the next section.

The structure of background metric in quantum theory of gravity is open question [15]. To analyze the structure of the background metric would be possible if we knew physical part of the effective action exactly. The minimum of the physical part determines the background metric. The background is to be found by solving the equation of motion for the effective action in terms of the dressed effective fields of matter sector and gravity sector. We will assume that the solution to such equations of motion gives Minkowski or AdS metric on the gravity side. However, metric perturbations about the background geometry in course of particle collisions can become strong and horizon forms.

### 3 Graviton scattering in four dimensions

Based on the construction of the previous section, let us begin with the pure four-dimensional gravity without any higher dimensional additions. Typical examples considered in literature concern collision of two shock waves in Aichelburg-Sexl gauge [17]. There are two physically different cases, both of which have been analyzed in references at the semiclassical level. First case is the scattering of one particle by a shock wave of another. Second case is gravitational collapse of two particles into black hole. The effective parameter to differentiate these two cases is fraction of the effective Schwarzschild radius to the impact parameter of the problem. In the first case the black hole does not appear.

Lipatov [18] has proposed to calculate scattering amplitudes of gravitons from some effective action which is local. This action restores unitarity which is lost in the leading approximation. According to Ref. [19] the eikonal amplitude can be calculated by solving equations of motion that come from the extremizing the Lipatov action and estimating the value of this action on the effective trajectories of motion. The action of Ref. [18] gives the same results for the tree level scattering amplitudes of gravitons. In Ref. [19] it has been checked that the Lipatov action reproduces eikonal amplitudes correctly.

The Lipatov effective action [18] is

$$\int d^4x L_{eff} = \int d^4x (L_0 + L_e + L_r), \quad (12)$$

where  $L_0$  is the kinetic term of relevant degrees of freedom,  $L_e$  is the graviton emission-absorption term,  $L_r$  is rescattering term, which is usually neglected [19]. The exact form of these terms is

$$L_0 = -2\partial h^{--}\partial^* h^{++} + \partial_+\partial^{*2} \phi \partial_-\partial^2 \phi^*. \quad (13)$$

Here  $h^{--}$ ,  $h^{++}$  are longitudinal degrees of freedom of the metric  $h^{\mu\nu}$ . They have been defined by using the only non-zero components of the energy-momentum tensor

$$T_{--} = kE\delta(x^-)\delta^2(\mathbf{x}), \quad T_{++} = kE\delta(x^+)\delta^2(\mathbf{x} - \mathbf{b}), \\ x^\pm = x^0 \pm x^3, \quad \mathbf{x} = (x^1, x^2), \quad z = x^1 + ix^2, \quad \partial^* = \partial/\partial z^*, \quad k^2 = 8\pi G,$$

in the following way

$$T_{\mu\nu}(x)h^{\mu\nu}(x) = T_{--}(x)h^{--}(x) + T_{++}(x)h^{++}(x),$$

where  $x$  is four-dimensional coordinate. The  $L_e$  is interaction Lagrangian. It is important for estimating two-loop correction. In order to estimate leading order correction  $L_e$  is not necessary. Equations of motion that come from the Lipatov action can be factorized [19] by the substitution

$$h^{++}(x) = kE\delta(x^-)a(z, z^*), \quad h^{--}(x) = kE\delta(x^+)\bar{a}(z, z^*) \\ \phi = \frac{1}{2}k(kE)^2\frac{1}{2}\theta(x^+x^-)\varphi(z, z^*) \\ \bar{a}(b-z, b-z^*) = a(z, z^*).$$

The physical picture depends on how large is impact parameter  $b = |\mathbf{b}|$  in comparison with the effective Schwarzschild radius  $R_S$ . If  $b$  is much larger than  $R_S$  the black hole does not appear and one has pure gravitational scattering process [19]. In the limit case  $b \gg R_S$ , the effective equations of motion are free field solutions which are Aichelburg-Sexl shock waves [20],

$$a(z, z^*) = -\frac{1}{2\pi} \log \frac{zz^*}{L^2}, \quad \varphi = 0, \quad (14)$$

where  $L$  is a large-distance scale, which plays the role of infrared cutoff. Moreover, the Lipatov action has reproduced in the next approximation with finite impact parameter  $b$  the result of direct calculations of Ref. [21] of the eikonal amplitude up to two-loop level [19]. The value of the effective action on the effective equations of motion (14) is [19]

$$A(a, \bar{a}, \varphi) = 2\pi Gs \left( \bar{a}(0) + a(b) + \int d^2x [\bar{a}\partial^*\partial a + a\partial^*\partial \bar{a}] \right) \\ = 2\pi Gs \frac{1}{2} (\bar{a}(0) + a(b)) = -2Gs \log \frac{b}{L},$$

which is phase of eikonal  $\delta(s, b)$ . Then the leading eikonal function is equal to [19]

$$S = \exp \left( -2\frac{i}{\hbar} Gs \log \frac{b}{L} \right). \quad (15)$$

Then by going from  $s, b$  representation to  $s, t$  representation one can write *two particles*  $\rightarrow$  *two particles* process scattering amplitude

$$\begin{aligned} a(s, b) &= e^{i\delta(s, b)}, \\ a(s, t) &= \frac{1}{(2\pi)^2} \int d^2x \, e^{iq_i x^i} e^{i\delta(s, b)} \\ &= \frac{1}{2\pi} \int_0^\infty db b J_0(bq) e^{i\delta(s, b)}. \end{aligned} \quad (16)$$

Taking into account (15), we can reproduce the 't Hooft result [20] for the amplitude,

$$a(s, t) \sim G \frac{s}{t}.$$

However, this is only scattering amplitude of two particles in Minkowski background geometry. When the scattering parameter is small, the black hole formation can occur. This case has been investigated in Refs. [22, 23, 24] for the case of axisymmetric ( $b = 0$ ) two black holes collision. The form of the trapped surface which appears in such a collision has been found there. The criteria of the black hole formation is appearance of this trapped surface [25]. The form of the trapped surface in four-dimensional collisions of two particles for  $b \neq 0$  has been found in Ref. [26]. It has been found there that the correction changes form of the geometrical cross-section a little by some factor of the order of 1.

## 4 Graviton scattering in RS2 case

The same idea can be applied to five-dimensional case. Scattering amplitudes can be considered as value of the effective action calculated on the solution to the equations of motion. Here we have Randall-Sundrum model with infinite fifth dimension (RS2 scenario). Emparan [27] has extended the analysis of the shock wave of an ultrahigh-energy particle to scenarios with extra dimensions. The metric of shock wave in five dimensions has the form

$$ds^2 = dy^2 + e^{-2y/R} (-dudv + dx^i dx^i + h_{uu}(u, x_i, y) du^2),$$

where  $u, v$  are light-cone coordinates and  $x_i$  are coordinates in the directions transverse to propagation,  $y$  is the coordinate in fifth dimension. The equations of motion for the shock wave addition to AdS space  $h_{uu}$  has been solved analytically [27] in the linearized approximation and then it has been argued that the solution to the linearized equation is in fact exact in this model [27]. The analysis analogous to the four-dimensional case can be applied again and one obtains that in case  $b > R$  the eikonal phase is

$$\delta(s, b) = -G_4 s \left( \log \frac{b}{R} - \frac{R^2}{2b^2} \right).$$

The second term is the Kaluza-Klein correction to the pure four-dimensional eikonal term,  $G_4$  is four-dimensional coupling of gravity.

In the limit  $r \ll R$  the solution for  $h_{uu}$  is [27]

$$h_{uu} = -4G_4 p \delta(u) \left[ -\frac{R}{r} + \frac{3}{2} \log(r/R) + \frac{3r}{8R} + \dots \right],$$



and this means that eikonal scattering phase is

$$\delta(s, b) = \frac{G_4 s R}{2b}.$$

In the  $s, t$  representation (16) the amplitude is [27]

$$a(s, t) \sim G_4 R \frac{s^2}{\sqrt{-t}}.$$

This is the case of scattering. Again, as in the four dimensional case one has to search for the trapped surface to trace the black hole formation. At present, the result for trapped surface are absent in literature. Some estimations of the shape of horizon for the point static source on the brane have been done by Giddings [6].

## 5 Giddings' estimation for FB in gravity with extra dimensions

As is suggested, there is a minimum of the effective action  $\Gamma$  in the form of AdS metric

$$ds^2 = dy^2 + e^{-2y/R} ds_M^2,$$

where  $ds_M^2$  is Minkowski interval and that equations of motion of two particles in this background are non-linearized Einstein equations. One can consider static source of perturbations from the background metric on the brane as product of collision. In linearized approximation we just follow by the idea of Giddings when estimating size of horizon by the requirement  $h_{00} \sim -1$ . The field describing the perturbation about background is related to the variable of the effective action whose derivatives with respect to that variable are vertices of gravitons in this background.

The metric of the theory in extra dimension is taken in the form of perturbed AdS metric. It can be considered as some solution in low energy action of string theory. The perturbed AdS metric takes the form [6]

$$ds^2 = (1 + h_{yy}) dy^2 + e^{-2y/R} (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu$$

where  $x$  are coordinates of our four dimensional world and  $y$  is a coordinate in fifth extra dimension. The energy-momentum tensor is a source localized on the IR brane,

$$T_{\mu\nu}(x, y) = S_{\mu\nu}(x) \delta(y), \quad T_{yy} = T_{y\mu} = 0.$$

The point mass source considered in [6] is

$$S_{\mu\nu}(x) = 2m \delta^{d-1}(x) \delta_\mu^0 \delta_\nu^0.$$

Linearized equations of motion can be solved by using Neumann Green's function

$$\square \Delta_{d+1}(x, y; x', y') = \frac{\delta^d(x - x') \delta(y - y')}{\sqrt{-G}}.$$

Under Neumann boundary conditions, the solution takes the form

$$h_{\mu\nu}(x, y) = -\frac{1}{2M_P^{d-1}} \int d^d x' \sqrt{-g} \Delta_{d+1}(x, y; x', 0) \left[ S_{\mu\nu}(x') - \eta_{\mu\nu} \frac{S_\lambda^\lambda(x')}{d-1} + \frac{\partial_\mu \partial_\nu S_\lambda^\lambda(x')}{\partial^2} \frac{1}{d-1} \right].$$

This behaviour of the metric fluctuations  $h_{\mu\nu}$  significantly simplifies at the large distances in four-dimensional space. The most interesting component  $h_{00}$  has the following behaviour

$$h_{00}(x, y) = -\frac{km}{RM_P^{d-1}} \frac{e^{-M_1 r + yd/R}}{r^{d-3}}.$$

Here  $k$  is a numerical constant and  $M_1 = C/R$ , where  $C$  is some number. This component of the metric is important since it is responsible for the horizon formation, when  $h_{00} = -1$ . Taking logarithm of the both parts, we come to the equation

$$M_1 r - yd/R = \ln \left[ \frac{km}{RM_P^{d-1} r^{d-3}} \right].$$

By solving this equation, one comes to the estimation of the area of the horizon in this model,

$$r_h(m) \sim \frac{1}{M_1} \ln \left[ \frac{kmM_1^{d-3}}{RM_P^{d-1}} \right].$$

Thus, the radius of horizon has logarithmic behavior in this brane model, in complete correspondence to the Froissart bound [6].

## 6 Conclusion

In this paper we have analysed general structure of the effective action for quantum gravity. We have shown that the effective action has the dressed effective fields as actual variables of the effective action. Kernels of the effective action written in terms of the dressed effective fields are constituent parts of the scattering amplitudes. In the eikonal approximation the scattering amplitudes can be found by solving the equations of motion derived from the Lipatov effective action.

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